

# Staggered Fermion Actions with Improved Rotational Invariance \*

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We introduce a class of improved actions for staggered fermions which to  $\mathcal{O}(p^4)$  and  $\mathcal{O}(p^6)$ , respectively, lead to rotationally invariant propagators. We discuss the resulting reduction of flavour symmetry breaking in the meson spectrum and comment on the improvement in the calculation of thermodynamic observables.

## 1. Introduction

In finite temperature calculations of QCD the cut-off dependence in the high temperature phase can be reduced with the  $\mathcal{O}(a^2)$  improved Naik action. The energy density of an ideal fermi gas only deviates by 20% from the continuum value on lattices with temporal extent  $N_\tau = 4$  to be compared with a deviation of 70% for the standard staggered action[1]. But still a further improvement is necessary to achieve an accuracy of a few percent as for the pure gauge calculations. Another lattice artifact is the breaking of flavour symmetry at  $\mathcal{O}(a^2)$  which is not improved in the Naik formulation[2]. An improvement by a factor of 2 can be achieved by introducing fat links in the standard fermion action[3].

## 2. Construction of fermion actions with improved rotational invariance

A general ansatz for a free fermion action with a difference scheme of arbitrary high order is given by

$$S_F = \sum_{x,\nu>\mu} \eta_{x,\mu} \bar{\psi}(x) \sum_{j>0,k,l,m} c_{j,k,l,m} \cdot \left[ \psi(x + ja_\mu + ka_\nu + la_\rho + ma_\sigma) - \psi(x - ja_\mu - ka_\nu - la_\rho - ma_\sigma) \right],$$

where  $j$  is odd and  $k, l, m$  are even to respect staggered symmetries.  $\eta_{x,\mu} = (-1)^{(x_0+\dots+x_{\mu-1})}$  and  $\eta_{x,0} = 1$  denote the staggered phases.

\*We thank the Deutsche Forschungsgemeinschaft for the support of the work under grants Pe 340/3-3, Pe 340/6-1

The coefficients can be fixed by demanding that the fermion propagator is rotationally invariant up to  $\mathcal{O}(p^4)$  or  $\mathcal{O}(p^6)$  respectively.

### • $\mathcal{O}(p^4)$ improved propagator

For actions including 1-link and 3-link paths in the difference scheme we obtain the constraints:

$$\begin{aligned} c_{1,0,0,0} + 3 c_{3,0,0,0} + 6 c_{1,2,0,0} &= 1/2 \\ c_{1,0,0,0} + 27 c_{3,0,0,0} + 6 c_{1,2,0,0} &= 24 c_{1,2,0,0} \end{aligned}$$

The fermion action  $S_F = \bar{\Psi} M \Psi$  then has the form

$$M[U]_{ij} = m \delta_{ij} + \eta_i \cdot \left( c_{1,0,0,0} A[U]_{ij} + c_{3,0,0,0} B_1[U]_{ij} + c_{1,2,0,0} B_2[U]_{ij} \right)$$

$$A[U]_{ij} = \sum_{\mu} \left( U_{i,\mu} \delta_{i-j-\hat{\mu}} - U_{i-\hat{\mu},\mu}^\dagger \delta_{i-j+\hat{\mu}} \right)$$

$$B_1[U]_{ij} = \sum_{\mu} \left( U_{i,\mu} U_{i+\hat{\mu},\mu} U_{i+2\hat{\mu},\mu} \delta_{i-j-3\hat{\mu}} - U_{i-\hat{\mu},\mu}^\dagger U_{i-2\hat{\mu},\mu}^\dagger U_{i-3\hat{\mu},\mu}^\dagger \delta_{i-j+3\hat{\mu}} \right)$$

$$\begin{aligned} B_2[U]_{ij} = & \sum_{\mu} \sum_{\nu \neq \mu} \left[ \left( U_{i,\mu} U_{i+\hat{\mu},\nu} U_{i+\hat{\mu}+\hat{\nu},\nu} \delta_{i-j-\hat{\mu}-2\hat{\nu}} \right. \right. \\ & - U_{i-\hat{\nu},\nu}^\dagger U_{i-2\hat{\nu},\nu}^\dagger U_{i-\hat{\mu}-2\hat{\nu},\mu}^\dagger \delta_{i-j+\hat{\mu}+2\hat{\nu}} \Big) \\ & + \left( U_{i,\nu} U_{i+\hat{\nu},\nu} U_{i+2\hat{\nu},\mu} \delta_{i-j-\hat{\mu}-2\hat{\nu}} \right. \\ & - U_{i-\hat{\mu},\mu}^\dagger U_{i-\hat{\mu}-\hat{\nu},\nu}^\dagger U_{i-\hat{\mu}-2\hat{\nu},\nu}^\dagger \delta_{i-j+\hat{\mu}+2\hat{\nu}} \Big) \\ & + \left( U_{i-\hat{\nu},\nu}^\dagger U_{i-2\hat{\nu},\nu}^\dagger U_{i-2\hat{\nu},\mu} \delta_{i-j-\hat{\mu}-2\hat{\nu}} \right. \\ & \left. \left. - U_{i-\hat{\mu},\mu}^\dagger U_{i-\hat{\mu},\nu} U_{i-\hat{\mu}+\hat{\nu},\nu} \delta_{i-j+\hat{\mu}-2\hat{\nu}} \right) \right] \end{aligned}$$

$$+ \left( U_{i,\mu} U_{i+\hat{\mu}-\hat{\nu},\nu}^\dagger U_{i+\hat{\mu}-2\hat{\nu},\mu}^\dagger \delta_{i,j-\hat{\mu}+2\hat{\nu}} - U_{i,\nu} U_{i+\hat{\nu},\nu}^\dagger U_{i-\hat{\mu}+2\hat{\nu},\mu}^\dagger \delta_{i,j+\hat{\mu}-2\hat{\nu}} \right) \Big]$$

Choosing  $c_{1,2,0,0} = 0$  yields the Naik action:

$$c_{1,0,0,0} = 9/16 \quad c_{3,0,0,0} = -1/48$$

Choosing the linear 3-link term to be zero we obtain the so-called p4 action:

$$c_{1,0,0,0} = 3/8 \quad c_{1,2,0,0} = 1/48$$

•  $\mathcal{O}(p^6)$  improved propagator:

For actions including up to 7-link paths of euclidean length up to  $\sqrt{13}$  in the difference scheme we find four equations which constrain the six coefficients.

In Table 1 we show two possible choices of the  $c_{i,j,k,l}$ , the first one, denoted by p6m, corresponds to the minimal set of four non-vanishing parameters, the second one, called p6, contains two free parameters which can be used for tuning. The coefficients of the fixed point action (fp) which are shown in the third column are similar to those of the rotational invariant actions i.e. the value of the coefficients is to a large extent determined by rotational invariance.

$(i, j, k, l)$	$c_{i,j,k,l}$		
	p6m	p6	fp [4]
(1,0,0,0)	0.3375	0.32	0.3308695
(1,2,0,0)	0.01875	0.02	0.0220591
(1,2,2,0)	0.0023438	0.0010938	0.0023284
(3,0,0,0)	0.0072917	0.0047917	0.0117443
(1,2,2,2)		0.00125	0.0002419
(3,2,0,0)		0.00125	-0.0002466
(....)			...

Table 1. Coefficients for  $\mathcal{O}(p^6)$  rotational invariant actions and fixed point action.

### 3. Thermodynamic properties and zero temperature dispersion relations

In pure gauge theory improvement of the high temperature behavior with tree level improved actions leads to a large improvement even close to  $T_c$ . It is expected that the same will hold for

full QCD when an improved fermion action with an improved high temperature limit is used.

In Figure 1 we show the free fermion energy density for various actions and note that the p4 action has deviations from the continuum value of maximum 8% (for  $N_\tau = 6$ ).

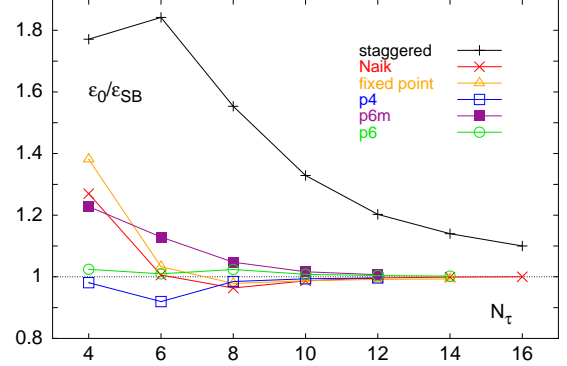


Figure 1. Deviations from the ideal gas value for various fermion actions.

The dispersion relation,  $E = E(p_x, p_y, p_z)$ , results from the poles of the propagator,  $D^{-1}(iE, \vec{p}) = 0$ . The p4 action dispersion relation shown in Figure 2 is close to the continuum form  $E(p) = p$  for a wide range of momenta.

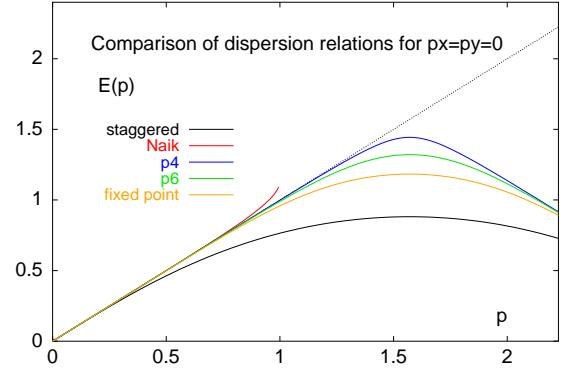


Figure 2. Zero temperature dispersion relations for several actions.

### 4. Testing flavour symmetry - Numerical results

We have performed a quenched simulation using a tree level improved  $1 \times 2$  action at  $\beta = 4.10$  on a  $16^3 \times 30$  lattice. We calculated the meson propagators from wall sources on 57 configura-

action	$\omega$	$m_\pi$	$m_{\pi_2}$	$m_\rho$	$m_\pi/m_\rho$	$\Delta_\pi$
staggered	0.0	0.598(1)	0.792(7)	0.979(15)	0.613(10)	0.199(8)
p4	0.0	0.643(1)	0.838(6)	1.015(15)	0.633(10)	0.192(7)
p6	0.0	0.676(1)	0.870(9)	1.032(15)	0.655(10)	0.188(9)
staggered	0.2	0.571(1)	0.645(3)	0.888(15)	0.643(11)	0.086(4)
staggered	0.4	0.569(1)	0.635(3)	0.865(15)	0.658(12)	0.076(4)
p4 <sub>fat1</sub>	0.2	0.619(1)	0.694(3)	0.915(20)	0.676(15)	0.082(4)
p4 <sub>fat2</sub>	0.2	0.609(1)	0.693(4)	0.905(20)	0.673(15)	0.093(5)

Table 2.  
Meson masses,  $\pi/\rho$   
mass ratio and pion  
splitting for various  
actions at a bare quark  
mass of  $m_q a = 0.05$ .

tions for several fermion actions including also fat links [3].

An indicator of flavour symmetry breaking is the mass splitting of the Goldstone pion  $\pi$  and the non-Goldstone pion  $\pi_2$  normalized by the rho meson mass,  $\Delta_\pi = (m_{\pi_2} - m_\pi) / m_\rho$ .

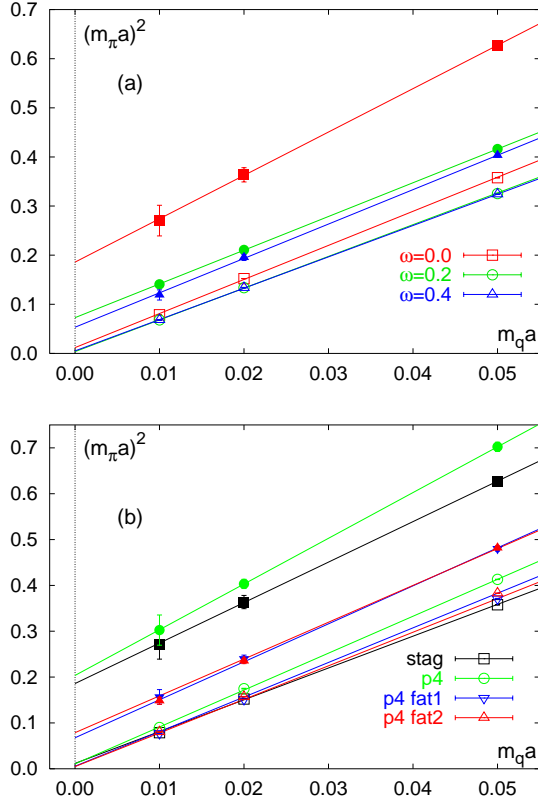


Figure 3. The squared pion mass vs the bare quark mass (a) for standard fat and non-fat actions and (b) for fat p4 and non-fat standard and p4 actions. Unfilled symbols are the Goldstone, filled symbols the non-Goldstone pions.

In Table 2 we show results of the pion splitting for the following actions with a bare quark mass  $m_q a = 0.05$  :

- standard staggered, p4 and p6 action
- standard staggered action with fat links for  $\omega = 0.2$  and  $\omega = 0.4$
- p4 action with fat links in the 1-link path (p4<sub>fat1</sub>) and p4 action with fat links in the 1-link and 3-link path (p4<sub>fat2</sub>) for  $\omega = 0.2$

One finds that for the p4 and p6 actions the pion splitting is not reduced significantly, whereas fat p4 actions lead to an improvement by a factor of 2 similar to what has been observed for the standard action. In Figure 3 we show the squared pion mass vs the bare quark mass with a linear extrapolation to zero quark mass. We do not see any significant improvement of flavour symmetry for the tree level improved gauge action compared to the Wilson gauge action. For the fat p4 action the improvement of flavour symmetry persists in the chiral limit.

## 5. Conclusions

The p4 action using fat links in the one-link term is a good candidate for finite temperature simulations in full QCD. Both thermodynamic properties and flavour symmetry are improved significantly.

## REFERENCES

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